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Racines carrées (Solution)

Exercice 1 :

* $\sqrt{98^2} = \boxed{98}$
* $(4\sqrt{3})^2 = 4^2 \times (\sqrt{3})^2 = 16 \times 3 = \boxed{48}$
* $-5(\sqrt{11})^2 = -5 \times 11 = \boxed{-55}$
* $\sqrt{(-8)^2} = \sqrt{8^2} = \boxed{8}$
* $\sqrt{100} - \sqrt{16} = \sqrt{10^2} - \sqrt{4^2} = 10 - 4 = \boxed{6}$
* $\sqrt{(169-25)^2} = \sqrt{144} = \sqrt{12^2} = \boxed{12}$
* $\sqrt{64+6^2} = \sqrt{64+36} = \sqrt{100} = \sqrt{10^2} = \boxed{10}$
* $\left(\frac{\sqrt{2025}}{\sqrt{2024}}\right)^2 = \frac{\sqrt{2025}^2}{\sqrt{2024}^2} = \boxed{\frac{2025}{2024}}$
* $\sqrt{3^2 \times 7^4} = 3\sqrt{7^4} = 3\sqrt{(7^2)^2} = 3 \times 7^2 = 3 \times 49 = \boxed{147}$
* $\sqrt{0,0081} = \sqrt{0,09^2} = \boxed{0,09}$
* $\sqrt{6^2+3^2+2^2} = \sqrt{36+9+4} = \sqrt{49} = \sqrt{7^2} = \boxed{7}$
* $\sqrt{\frac{0,49 \times 0,09}{0,81 \times 0,36}} = \sqrt{\frac{0,7^2 \times 0,3^2}{0,9^2 \times 0,6^2}} = \frac{0,7 \times 0,3}{0,9 \times 0,6} = \frac{0,21}{0,54} = \frac{21}{54} = \boxed{\frac{7}{18}}$

Exercice 2 :

* $\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = \sqrt{6^2} = \boxed{6}$
* $\sqrt{7} \times \sqrt{28} = \sqrt{7 \times 28} = \sqrt{196} = \sqrt{14^2} = \boxed{14}$
* $\sqrt{19} \times \sqrt{76} = \sqrt{19 \times 76} = \sqrt{1444} = \sqrt{38^2} = \boxed{38}$
* $\sqrt{50} \times \sqrt{\frac{1}{2}} = \sqrt{50 \times \frac{1}{2}} = \sqrt{25} = \sqrt{5^2} = \boxed{5}$
* $\sqrt{\frac{9}{10}} \times \sqrt{\frac{40}{81}} = \sqrt{\frac{9 \times 40}{10 \times 81}} = \sqrt{\frac{3^2 \times 4 \times 10}{10 \times 9^2}} = \sqrt{\frac{3^2 \times 2^2}{9^2}} = \frac{3 \times 2}{9} = \boxed{\frac{2}{3}}$
* $\sqrt{55} \times \sqrt{33} \times \sqrt{15} = \sqrt{55 \times 33 \times 15} = \sqrt{5 \times 11 \times 3 \times 11 \times 3 \times 5} = \sqrt{5^2 \times 3^2 \times 11^2}$
$= 5 \times 3 \times 11 = \boxed{165}$

Exercício 3:

* $\frac{2}{\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3}^2} = \boxed{\frac{2\sqrt{3}}{3}}$

* $\frac{-\sqrt{5}}{\sqrt{2}} = \frac{-\sqrt{5} \times \sqrt{2}}{(\sqrt{2})^2} = \boxed{\frac{-\sqrt{10}}{2}}$

* $\frac{1+\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}(1+\sqrt{7})}{(\sqrt{7})^2} = \frac{\sqrt{7}(1+\sqrt{7})}{7} = \frac{\sqrt{7} \times \sqrt{7}(1+\sqrt{7})}{(\sqrt{7})^2} = \boxed{\frac{\sqrt{7}\sqrt{7}(1+\sqrt{7})}{7}}$

* $\frac{2\sqrt{2}-3}{2\sqrt{2}+3} = \frac{(2\sqrt{2}-3)^2}{(2\sqrt{2}+3)(2\sqrt{2}-3)} = \frac{(2\sqrt{2})^2 - 2 \times 2\sqrt{2} \times 3 + 3^2}{(2\sqrt{2})^2 - 3^2} = \frac{4 \times 2 - 12\sqrt{3} + 3}{4 \times 2 - 9}$

$= \frac{8 - 12\sqrt{3} + 3}{8 - 9} = \frac{11 - 12\sqrt{3}}{-1} = \boxed{12\sqrt{3} - 11}$

* $\frac{\sqrt{3}-\sqrt{5}}{2+\sqrt{5}} = \frac{(\sqrt{3}-\sqrt{5})(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{\sqrt{3} \times 2 - \sqrt{3} \times \sqrt{5} - \sqrt{5} \times 2 + (\sqrt{5})^2}{2^2 - (\sqrt{5})^2}$

$= \frac{2\sqrt{3} - \sqrt{15} - 2\sqrt{5} + 5}{4 - 5} = \frac{2\sqrt{3} - \sqrt{15} - 2\sqrt{5} + 5}{-1} = \boxed{\sqrt{15} + 2\sqrt{5} - 2\sqrt{3} - 5}$

* $\frac{\sqrt{48} - 2\sqrt{75}}{\sqrt{27} + \sqrt{12}} = \frac{\sqrt{16 \times 3} - 2\sqrt{25 \times 3}}{\sqrt{9 \times 3} + \sqrt{4 \times 3}} = \frac{4\sqrt{3} - 10\sqrt{3}}{3\sqrt{3} + 2\sqrt{3}} = \frac{-6\sqrt{3}}{5\sqrt{3}} = \boxed{\frac{-6}{5}}$

* $\frac{2}{1-\sqrt{2}+\sqrt{3}} = \frac{2}{(1-\sqrt{2})+\sqrt{3}} = \frac{2[(1-\sqrt{2})-\sqrt{3}]}{[(1-\sqrt{2})+\sqrt{3}][(1-\sqrt{2})-\sqrt{3}]}$

$= \frac{2(1-\sqrt{2}-\sqrt{3})}{(1-\sqrt{2})^2 - (\sqrt{3})^2} = \frac{2(1-\sqrt{2}-\sqrt{3})}{1 - 2\sqrt{2} + 2 - 3} = \frac{2(1-\sqrt{2}-\sqrt{3})}{-2\sqrt{2}}$

$= \frac{2\sqrt{2}(1-\sqrt{2}-\sqrt{3})}{-2(\sqrt{2})^2} = \frac{2\sqrt{2} - 2 \times 2 - 2\sqrt{6}}{-2 \times 2} = \frac{2\sqrt{2} - 2\sqrt{6} - 4}{-4}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2} + 4}{2}}$

* $\frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} = \frac{(\sqrt{2}-\sqrt{3}-\sqrt{5})[(\sqrt{2}+\sqrt{3})-\sqrt{5}]}{[(\sqrt{2}+\sqrt{3})+\sqrt{5}][(\sqrt{2}+\sqrt{3})-\sqrt{5}]}$

$= \frac{(\sqrt{2}-\sqrt{3}-\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2 - (\sqrt{5})^2}$

$= \frac{2 + \sqrt{6} - \sqrt{10} - \sqrt{6} - 3 + \sqrt{15} - \sqrt{10} - \sqrt{15} + 5}{2 + 2\sqrt{6} + 3 - 5}$

$= \frac{4 - 2\sqrt{10}}{2\sqrt{6}} = \frac{2 - \sqrt{10}}{\sqrt{6}} = \frac{\sqrt{6}(2-\sqrt{10})}{(\sqrt{6})^2} = \frac{2\sqrt{6} - \sqrt{60}}{6}$

$$= \frac{2\sqrt{6} - 2\sqrt{15}}{6} = \boxed{\frac{\sqrt{6} - \sqrt{15}}{3}}$$

Exercice 4 :

* $A = \sqrt{\sqrt{49} + 2} - 3\sqrt{25} = \sqrt{\sqrt{7^2} + 2} - 3\sqrt{5^2} = \sqrt{7+2} - 3\times 5 = \sqrt{9} - 15$

$= \sqrt{3^2} - 15 = 3 - 15 = \boxed{-12}$

* $B = \sqrt{7+2\sqrt{6}} \times \sqrt{7-2\sqrt{6}} = \sqrt{(7+2\sqrt{6})(7-2\sqrt{6})} = \sqrt{7^2 - (2\sqrt{6})^2} = \sqrt{49 - 4\times 6}$

$= \sqrt{49 - 24} = \sqrt{25} = \sqrt{5^2} = \boxed{5}$

* $C = \sqrt{20 - 8\sqrt{2}} \times \sqrt{45 + 18\sqrt{2}} = \sqrt{4(5 - 2\sqrt{6})} \times \sqrt{9(5 + 2\sqrt{6})}$

$= \sqrt{2^2(5 - 2\sqrt{6})} \times \sqrt{3^2(5 + 2\sqrt{6})} = 2\sqrt{5 - 2\sqrt{6}} \times 3\sqrt{5 + 2\sqrt{6}}$

$= 6\sqrt{(5 - 2\sqrt{6})(5 + 2\sqrt{6})} = 6\sqrt{5^2 - (2\sqrt{6})^2} = 6\sqrt{25 - 4\times 6}$

$= 6\sqrt{25 - 24} = 6\sqrt{1} = \boxed{6}$

* $D = \sqrt{2}\sqrt{\sqrt{10} - \sqrt{2}} \times \sqrt{\sqrt{10} + \sqrt{2}} = \sqrt{2}\sqrt{(\sqrt{10} - \sqrt{2})(\sqrt{10} + \sqrt{2})}$

$= \sqrt{2}\sqrt{\sqrt{10}^2 - \sqrt{2}^2} = \sqrt{2}\sqrt{10 - 2} = \sqrt{2} \times \sqrt{8} = \sqrt{2\times 8} = \sqrt{16} = \boxed{4}$

Exercice 5 :

1) $A^2 = \left(\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}\right)^2$

$= \sqrt{7+4\sqrt{3}}^2 + 2 \times \sqrt{7+4\sqrt{3}} \times \sqrt{7-4\sqrt{3}} + \sqrt{7-4\sqrt{3}}^2$

$= 7 + 4\sqrt{3} + 2\sqrt{(7+4\sqrt{3})(7-4\sqrt{3})} + 7 - 4\sqrt{3}$

$= 14 + 2\sqrt{7^2 - (4\sqrt{3})^2}$

$= 14 + 2\sqrt{49 - 16\times 3}$

$= 14 + 2\sqrt{49 - 48}$

$= 14 + 2\sqrt{1}$

$= 14 + 2$

$= \boxed{16}$

2) D'après la question 1), on a $A^2 = 16$, or A est un nombre positif, donc $A = \sqrt{16} = 4$. D'où $\boxed{A = 4}$

Exercice 6 :

1) * $A^2 = (\sqrt{5}+\sqrt{3})^2 = \sqrt{5}^2 + 2\times\sqrt{5}\times\sqrt{3} + \sqrt{3}^2 = 5 + 2\sqrt{15} + 3 = \boxed{8 + 2\sqrt{15}}$

* $B^2 = (\sqrt{5}-\sqrt{3})^2 = \sqrt{5}^2 - 2\sqrt{5}\times\sqrt{3} + \sqrt{3}^2 = 5 - 2\sqrt{15} + 3 = \boxed{8 - 2\sqrt{15}}$

* $A\times B = (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = \sqrt{5}^2 - \sqrt{3}^2 = 5 - 3 = \boxed{2}$

2) On a : $\frac{A}{B} + \frac{B}{A} = \frac{A^2+B^2}{AB}$

$= \frac{8 + 2\sqrt{15} + 8 - 2\sqrt{15}}{2}$ (D'après la question 1)

$= \frac{16}{2}$

$= 8$

D'où $\frac{A}{B} + \frac{B}{A}$ est un entier.

3) On a : $\sqrt{14+6\sqrt{5}} = \sqrt{9+6\sqrt{5}+5}$

$= \sqrt{3^2 + 2\times 3\times\sqrt{5} + \sqrt{5}^2}$

$= \sqrt{(3+\sqrt{5})^2}$

$= \boxed{3+\sqrt{5}}$

Exercice 7 :

1) On a : $(\sqrt{a}+\sqrt{b})^2 + (\sqrt{a}-\sqrt{b})^2 = \sqrt{a}^2 + 2\times\sqrt{a}\times\sqrt{b} + \sqrt{b}^2 + \sqrt{a}^2 - 2\sqrt{a}\times\sqrt{b} + \sqrt{b}^2$

$= a + 2\sqrt{ab} + b + a - 2\sqrt{ab} + b$

$= 2a + 2b$

$= \boxed{2(a+b)}$

2) On a : $(\sqrt{a}+\sqrt{b})^2 - (\sqrt{a}-\sqrt{b})^2 = \sqrt{a}^2 + 2\times\sqrt{a}\times\sqrt{b} + \sqrt{b}^2 - (\sqrt{a}^2 - 2\sqrt{a}\times\sqrt{b} + \sqrt{b}^2)$

$= a + 2\sqrt{ab} + b - (a - 2\sqrt{ab} + b)$

$= a + 2\sqrt{ab} + b - a + 2\sqrt{ab} - b$

$= \boxed{4\sqrt{ab}}$

Exercice 8:

On a: $E = \sqrt{10+4\sqrt{6}} - \sqrt{15+6\sqrt{6}}$

$= \sqrt{4+4\sqrt{6}+6} - \sqrt{9+6\sqrt{6}+6}$

$= \sqrt{2^2+2\times2\times\sqrt{6}+\sqrt{6}^2} - \sqrt{3^2+2\times3\times\sqrt{6}+\sqrt{6}^2}$

$= \sqrt{(2+\sqrt{6})^2} - \sqrt{(3+\sqrt{6})^2}$

$= 2+\sqrt{6}-(3+\sqrt{6})$

$= 2+\sqrt{6}-3-\sqrt{6}$

$= \boxed{-1}$

Exercice 9:

a et b sont deux nombres réels positifs tels que $a-\sqrt{b}=4$ et $a^2-b=20$.

Calculons ab.

On a: $a^2-b=20$

S.q: $a^2-\sqrt{b}^2=20$

S.q: $(a-\sqrt{b})(a+\sqrt{b})=20$

Or: $a-\sqrt{b}=4$, donc: $4(a+\sqrt{b})=20$ S.q: $a+\sqrt{b}=\frac{20}{4}=5$

On a donc $\boxed{a-\sqrt{b}=4}$ (1) et $\boxed{a+\sqrt{b}=5}$ (2)

On additions les membres des deux égalités (1) et (2), on obtient:

$a-\sqrt{b}+a+\sqrt{b}=4+5$

S.q: $2a=9$ et donc $\boxed{a=\frac{9}{2}}$ (*)

Or: $a-\sqrt{b}=4$ et $a=\frac{9}{2}$, donc: $\frac{9}{2}-\sqrt{b}=4$

S.q: $-\sqrt{b}=4-\frac{9}{2}$

S.q: $-\sqrt{b}=-\frac{1}{2}$

S.q: $\sqrt{b}=\frac{1}{2}$

S.q: $\boxed{b=\frac{1}{4}}$ (**)

De (*) et (**), on trouve que: $ab=\frac{9}{2}\times\frac{1}{4}=\frac{9}{8}$

Par suite: $\boxed{ab=\frac{9}{8}}$